Comparison of Calibration Algorithms for Estimation of Rainfall Using Weather Radar

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Abstract: The use of data from a weather radar network is an efficient way of observing the structure and behavior of a rainfall field. A weather radar can provide spatially and temporally continuous measurements covering a large area, that can be used almost simultaneously as the storm occurs. However, it is widely recognized that the algorithms used to estimate rainfall based on radar observations have a high degree of uncertainty. This uncertainty may be caused by the variability of rainfall drop size distribution, the variation of reflectivity with height and with range, the temporal and spatial resolutions adopted for sampling the radar reflectivity, and radar hardware miscalibration and noise. Therefore, the use of radar-derived rainfall products in hydrological applications requires a proper specification of the relationship between radar reflectivity and rainfall rate. The aim of this paper is to study the methods used to estimate rainfall based on reflectivity, identify scenarios where the method may be found wanting, and develop alternatives that are able to remove the identified deficiencies. Two rainfall estimation algorithms are examined. The first involves specification of a theoretically prescribed parametric relationship, and the second attempts to match rainfall quantiles to reflectivity quantiles for the same exceedence probability. A new approach that combines certain aspects of the above two methods is formulated. Synthetic rainfall-reflectivity data are used to test the efficiency and applicability of all three rainfall estimation methods. A 6-month rainfall-reflectivity record from the Kurnell radar at Sydney is used to illustrate the applicability of the proposed method to real data.

Keywords: Weather radar; Rainfall; Calibration

1. INTRODUCTION

The frequency, accuracy and resolution of hydrologic records is a major limitation in the accurate modelling of hydrologic events. The advent of the weather radar has provided the means for measuring rainfall continuously at fine spatial and temporal resolutions. However, considerable uncertainty still remains in the procedures used to estimate the rainfall from weather radar observations. This paper discusses the procedures that are currently in use for estimation of radar rainfall, highlighting their advantages and limitations, and proposes a new approach that reduces some of the uncertainty involved.

A weather radar measures the power reflected back by raindrops, and uses this as the basis of estimating rainfall intensity. The backscattered power is proportional to the reflectivity (Z), which is often related to the rainfall rate (R) based on the following parametric relationship:

$$Z = AR^B \tag{1}$$

Where A and B are constants estimated using actual raingauge observations. It is important to note that the above relationship e.q.(1) is valid under some rather stringent assumptions which do not hold true in real situations. As a result, significant differences are possible when rainfall estimated from radar reflectivity is compared to rainfall measurements using a network of raingauges.

The most common and widely used procedure for estimating rainfall from a weather radar is the non-linear regression fitting of simultaneous radar-reflectivity observations (Z) and raingauge rainfall measurements (R) . As the "true" Z-R relationship depends on many factors which include the nature of the rainfall drop-size distribution, the appropriate values of A and B are dependent upon the type of precipitation captured by the radar beam. Empirical Z-R relationships and the variations from storm to storm and within individual storms have been studied extensively over the part 50 years [Joss and Waldvogel, 1990].

Another algorithm for estimating rainfall from radar observations is the probability matching method (PMM), proposed by Rosenfeld et al. [1993]. The rationale behind the PMM method is the fact that observed reflectivity (Z) is often very different from the true reflectivity (Z') near the surface, which would be better related to the rainfall as dictated by e.q. (1). The difference between observed and true reflectivity causes high variability in the fitted Z-R relationship. The PMM approach attempts to account for all such differences by estimating the rainfall such that its probability distribution is exactly the same as that for the radar reflectivity. In a statistical sense, the PMM method is matching the marginal probability distributions of the reflectivity and rainfall rate variables, without considering the joint distribution or inter-relationship of these variables. While this is not a major disadvantage in many situations, such an approach will give an inaccurate answer when there are hotspots, climatological noise, or range dependent biases in the actual radar data.

Rosenfeld et al. [1994] tried to moderate this effect by forming pairs of space-time windows small enough to maintain some physical relevance. This modified method is called Window PMM (WPMM). An advantage of WPMM is that the selected Z-R relationship is related to the variability associated with radar range and other parameters. However, the number of rainfall-reflectivity pairs used in the analysis are reduced.

To accommodate the effect of some of the physical factors, Ciach et al. [1997] proposed a conceptually different Z-R calibration procedure. This procedure conceptualises the calibration problem as an integrated optimisation of the whole radar rainfall algorithm. The algorithm used global optimisation with an objective function which minimises the root mean square error (RMSE) between radar rainfall and raingauge rainfall of the final product.

A different form of the parametric Z-R relationship has been proposed by Anagnostou [1997]. He proposed an adaptive global optimisation of six main parameters in real time, aimed at reducing the effect of range and storm type in the radar reflectivity observations. This procedure can simultaneously estimate all the parameters by optimising a criterion which minimises differences between the observed and estimated raingauge rainfall. Application of this procedure to two months of radar data from Melbourne, Florida shows satisfactory convergence of the model's parameters [Anagnostou et al., 1999].

As the calibration and validation of radar estimated rainfall are based upon the raingauge network, differences in measured rainfall between any two rainfall measuring sensors, even though they might be located close to each other, causes difficulties in formulating consistent Z-R relationships. In a statistical sense, such differences are equivalent to a significant amount of noise being present in the radar estimated rainfall. Consequently, no optimal Z-R relationship that is able to remove all the noise, can be expected to exist [Seed et al., 1997].

A new radar rainfall estimation approach is proposed in this paper. This approach attempts to reduce the uncertainty in the estimated rainfall by combining certain aspects of a prescribed parametric relationship with the PMM method. The rationale behind the approach is to formulate an altered measure of reflectivity that can be expected to have a monotonic relationship with rainfall, which can then be used in a PMM type approach. What follows is a description of the algorithm for the proposed method. This is followed by application to a synthetically generated rainfall-reflectivity dataset to illustrate the utility of the method under specific circumstances. We conclude with an application of the method to 6-months of rainfall-reflectivity data from the Kurnell radar in Sydney, Australia.

2. PROPOSED ALGORITHM

Some sources of uncertainty in the Z-R relationship are:

- Observed reflectivity is often very different from the true reflectivity due to the averaging of the real reflectivity field by the beam.
- The effects of such averaging are compounded because of necessity of comparing grid-averaged reflectivities with point raingauge measurements.
- Observed reflectivity can also be different as a function of distance or range due to increases in size and elevation of the radar sampling volume.
- Another important source of such difference is the variation in raindrop size distribution (DSD) between pulse volume (which is at a higher altitude) and the surface.

Unfortunately the first two factors cannot easily be reduced or removed, and have to be accepted as a source of white noise in radar reflectivity measurements. But the uncertainty caused by the last two factors might be reduced using appropriately formulated corrections. For example, one may use a relationship that corrects for systematic biases in reflectivity introduced because of increased distances from the radar, which in-turn causes the radar beam to scan at elevations that are greater than the elevation for cells that lie close proximity to the radar. Similarly, one may use a relationship that transforms the measured reflectivity to a variable that can be considered to be independent of changes in rainfall drop-size distributions, or, changes from one storm type to another. Once such transformations have been performed, what is left could be treated as noise in formulating rainfall estimation algorithms. While one approach to estimate rainfall from such a transformed reflectivity field would be to formulate a parametric relationship between the variables, as is done in e.q.(1), a more appropriate approach would be to match same probability quantiles of the two variables involved (this being the rationale of the PMM approach mentioned earlier). Such an approach will be superior to the prescription of a parametric relationship in all situations where the parametric relation is not fully appropriate.

The proposed method for estimating rainfall would work as follows:

- Transform measured reflectivity (Z) to a different variable (Z') that can be expected to have a consistent relationship with rainfall irrespective of differences due to distance from radar, or differences in the storm type.
- Formulate empirical cumulative distribution functions (CDFs) for the transformed reflectivity and measured rainfall.
- Estimate rainfall for new storms by first estimating the transformed reflectivity (Z'), then estimating its exceedence probability, which is then used to estimate a rainfall value that has the same exceedence probability.

Both synthetic data and the 6-month rainfall-reflectivity data record from the Kurnell radar at Sydney have been used to illustrate the efficiency and applicability of the existing radar rainfall estimation algorithms and the newly proposed algorithm.

3. TESTING THE EFFICIENCY AND APPLICABILITY OF RADAR ESTIMATED RAINFALL ALGORITHMS (SYNTHETIC DATA)

Synthetically generated rainfall-reflectivity data were generated using a method described in Seed et al. [1997]. Synthetic time series of rainfall intensity were generated using a lognormal distribution with mean of 0.45 and standard deviation of 0.45. This distribution was used in a case study of orographic rainfall in the foothills of the Southern Alps of New Zealand [Seed et al., 1997]. Corresponding reflectivity values were formulated so as to have a range-dependent bias, using the following relationship:

$$Z_t = A\left(\frac{R_t}{b_c\left(1 + arc * r / S_0\right)}\right)^B * \varepsilon_t \qquad (2)$$

where A and B are the Z-R relationship parameters which correspond to the conventional Z-R relationship, b_c is a parameter which differentiates between convective and stratiform cloud types [Tokay and Short, 1996], the parameter *arc* control the range correction applied on the rainfall estimates for the convective storms, S_0 is assumed to be 200 km, representing the effective radar range for rainfall estimation, and ε_t is the noise term, which has been generated by using a lognormal distribution with zero mean and different standard deviations that define the noise level in the resulting data.

3.1 Radar Rainfall Calibration

A 500 hour long synthetic time series of rainfall and reflectivity corresponding to a network of 28 raingauges was generated and used for calibration of radar rainfall. The noise term ε_t was generated such that the log-transformed noise had standard deviations equal to 0.1, 0.2 and 0.3.

To investigate the accuracy of the radar rainfall estimation algorithms, the R-square of each rainfall estimation algorithm was measured using e.q. (3). An R-square of one indicates that the estimated radar rainfall is a perfect fit to the raingauge rainfall, while a zero R-square implies otherwise. The estimated radar rainfall has lower accuracy than the mean of raingauge rainfall when R-square is less than zero.

$$R^2 = 1 - \frac{\sum_{i=1}^{N_t}\sum_{j=1}^{N_g}(R_g - R_r)^2}{\sum_{i=1}^{N_t}\sum_{j=1}^{N_g}(R_g - \overline{R}_g)^2} \qquad (3)$$

where R_g is the raingauge accumulation at the *j*th gauge, R_r is the radar accumulation around this gauge, both for the *i*th time period, N_g is the number of raingauges, N_t is the number of time periods and $\overline{R}_g$ is the mean of raingauge rainfall.

Altogether, five plausible rainfall estimation algorithms were studied:

- *Algorithm 1:* Use default parameter values of the Z-R relationship (A = 200 and B = 1.6).
- *Algorithm 2:* Use PMM method by matching the probability of exceedance between generated reflectivity and raingauge rainfall.
- *Algorithm 3:* Estimate A and B for the Z-R relationship by using an optimisation approach with the objective of minimizing RMSE between radar and raingauge accumulation as shown in e.q. (4).

$$RMSE = \left\{ \frac{1}{N_t N_g} \sum_{i=1}^{N_t} \sum_{j=1}^{N_g} [R_r(i,j) - R_g(i,j)]^2 \right\}^{1/2} \qquad (4)$$

- *Algorithm 4:* Use optimisation approach associated with range effect correction scheme to estimate A, B, b_c and arc parameter of the Z-R relationship. The objective of this optimisation algorithm is the same as algorithm 3.
- *Algorithm 5:* Combination of prescribed parametric relationship and PMM method by matching the CDF of transformed reflectivity and raingauge rainfall (Proposed algorithm).

The calculations in the algorithm 5 involve the following steps:

Step1: Estimate transformed reflectivity (Z') by using the prescribed parametric relationship.
Step2: Calculate the CDF of the transformed reflectivity obtained from step1, and the CDF of raingauge rainfall.
Step3: Estimate the final radar rainfall based on matching the CDFs of the transformed reflectivity and the raingauge rainfall obtained from step 2.

The R-square values that were obtained for the synthetic radar estimated rainfall corresponding to the different noise levels for each algorithm are presented in Table 1.

Table 1. Results: R-square for radar rainfall calibration.

Algorithm	Standard Deviation of ε_t		
	0.1	0.2	0.3
1	-0.027	0.052	0.017
2	0.805	0.781	0.716
3	0.806	0.784	0.724
4	0.818	0.793	0.738
5*	0.973	0.874	0.790

* Proposed algorithm

3.2 Radar Rainfall Validation

Radar rainfall validation has been carried out in order to test the applicability of the radar estimated rainfall algorithms when the parameters of the Z-R relationship have not been estimated in real time. Two different storm structures were used for radar rainfall validation. The first storm has the same structure as the calibration and second storm has different structure. The parameters obtained from the calibration of algorithm 1, 3 and 4 and the matching CDF of the generated reflectivity (algorithm 2)/transformed reflectivity (algorithm5) and raingauge rainfall were used to estimate the final radar rainfall for validation to both storm structures. Table 2. present the R-square for the two validation cases.

Table 2. Results: R-square for radar rainfall validation.

Algorithm	Standard Deviation of ε_t		
	0.1	0.2	0.3
1	-0.028/-1.234	0.011/-1.219	0.019/-1.162
2	0.787/-0.008	0.746/-0.017	0.661/-0.045
3	0.762/0.075	0.742/-0.078	0.635/0.053
4	0.811/0.065	0.779/0.08	0.730/0.044
5*	0.914/-0.027	0.870/-0.017	0.802/-0.054

xxx/yyy = same storm/different storm

3.3 Discussion of Results

The above calibration and validation results can be summarised as follows:

- ***Calibration:*** Table 1 shows that the optimisation approach associated with range effect correction scheme gives the most accurate radar rainfall (highest R-square) when compared to the existing radar rainfall estimation methods. Algorithm 2 (PMM) can improve R-square significantly compared to the use of default parameters (algorithm 1). This improvement occurs because the CDF of reflectivity is forced to attain the same shape as the CDF of raingauge rainfall.

From the calibration results it is found that the proposed algorithm (algorithm 5) gives the highest R-square compared to other methods. This is because the CDF of a transformed reflectivity that was free from range dependent biases was used to match with the CDF of raingauge rainfall. This result corresponds to the fact that the transformed reflectivity or equivalently, the unbiased radar rainfall, has the same distributional characteristics as the measured rainfall.

- ***Validation:*** The proposed algorithm gives a higher R-square than using the prescribed parametric relationship alone when validation is made on the same storm structure (see Table 2).

In most of the cases, the R-square decreases when the noise level increases. The R-square presented in the above table shows that all of the algorithms give inaccurate radar estimated rainfall when using different storm structure for validation. These results illustrate the simple fact that one needs to have records for calibration that reflect the type of events that could be expected to occur in future. If that is the case, the choice of the algorithm used should not affect the accuracy of the results obtained.

4. TESTING THE EFFICIENCY AND APPLICABILITY OF RADAR ESTIMATED RAINFALL ALGORITHMS (REAL DATA)

The 6-month 1.5 km CAPPI (Constant Altitude Plan Position Indicator) reflectivity data record from the Kurnell radar at Sydney and 89 hourly raingauges rainfall obtained from ALERT stations located within 150 km of the radar (as illustrated in Figure 1.) were used in this study. The reflectivity data are in Cartesian grids with 256 x 256 km extent and 1 km, 10 minute resolution. Storm classification has been carried out by considering both radar images and average rainfall intensity from the raingauge network. Storms were classified into widespread and convective types, the former usually coinciding with light rain and the latter with heavy rain. A non-quantitative visual interpretation was used in identifying the storm types. Storms that occurred during November 2000 to April 2001 were selected. These included three convective storms and six widespread events.

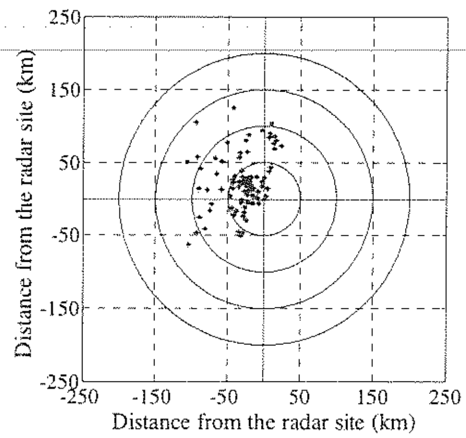

Figure 1. The spatial extent of the Kurnell Radar and Sydney raingauge network. (Circles correspond to 50 km, 100 km, 150 km and 200 km radar range. The Y axis corresponds to the north direction.)

As in the synthetic case, the proposed algorithm will give the best results when the final radar rainfall has been estimated from the matching of unbiased radar reflectivity with raingauge rainfall CDF's. Range dependent effects and different storm types have been considered as factors that cause biases in radar estimated rainfall.

4.1 Calibration

A plot of the average reflectivity and average rainfall intensity with range from the radar site (Figure 2), found that the trend of the average reflectivity with range corresponds to the average rainfall intensity. Therefore, in this study we consider that there is no range dependent bias in the observed reflectivity data. Hence, we assumed that only the biases due to different storm types remain in the observed reflectivity data. To eliminate these biases, the radar rainfall has been calibrated separately for convective and widespread storms. The calibration has been performed at an hourly time step. Quality control of rainfall-reflectivity data for the analysis has been carried out by using only the rainfall amounts which are greater than 0.3 mm/h, from raingauges where the correlation coefficient between raingauge rainfall and the corresponding reflectivity are higher than 0.3 when all storms are considered together. The Z-R relationship parameters have been estimated by using an optimisation scheme which minimises the RMSE between radar rainfall and raingauge rainfall.

It should be noted that since the calibration has been performed separately for each storm type, and since there are no corrections for range dependent bias in the observed reflectivity, the proposed algorithm collapses to the PMM approach. Three calibration strategies were tested. Firstly the prescribed parametric relationship, secondly the PMM method, and lastly the use of default parameters to estimate radar rainfall. To investigate the performance of PMM when applied to all storm types, all nine storms have been used together for calibration. The calibration results are presented in Table 3.

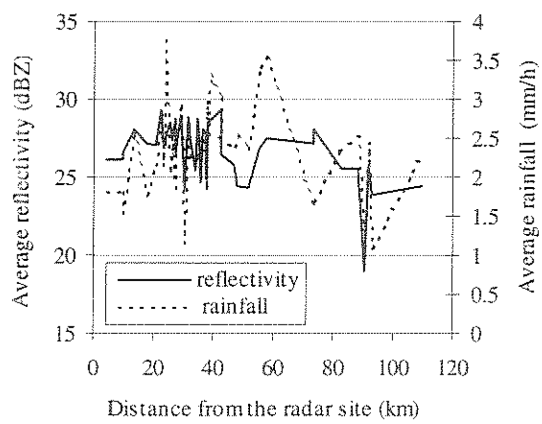

Figure 2. Trend of average reflectivity and rainfall intensity with range (using all 9 storms).

Table 3. Calibration results.

Convective 1172 Z-R pairs	A	B	RMSE (mm)	R^2
Prescribed parameters	74.16	1.76	3.06	0.453
PMM	-	-	3.31	0.361
Default parameters	200	1.6	3.32	0.356
Widespread 562 Z-R pairs	**A**	**B**	**RMSE (mm)**	**R^2**
Prescribed parameters	81.25	1.62	1.84	0.416
PMM	-	-	1.90	0.379
Default parameters	200	1.6	2.05	0.272
All storms (1828 Z-R pairs)	**A**	**B**	**RMSE (mm)**	**R^2**
Prescribed parameters	77.06	1.75	2.89	0.397
PMM	-	-	3.10	0.304
Default parameters	200	1.6	3.09	0.308

4.2 Discussion of Results

From the calibration results, it is evident that storm type affects the parameters of the Z-R relationship. The RMSEs obtained from the prescribed parametric relationship are better than those from PMM, for both storm types. This is to be expected unless the prescribed relation is substantially different to the true, as the parameters A and B are estimated to minimise the resulting error. A validation exercise is needed to confirm the utility of the estimated coefficients.

5. CONCLUSIONS

The efficiency and applicability of radar rainfall estimation algorithms, based on synthetically generated rainfall, synthetic reflectivity data, and real data, can be summarised as follows.

- A parametric relationship between reflectivity and rainfall is valid as long as it can reflect the uncertainties associated with the measured reflectivity values.
- The combination of the prescribed parametric relationship and PMM method will give the most improvement in radar rainfall estimation when the final radar rainfall has been estimated from matching of the unbiased radar reflectivity with raingauge rainfall CDF's.
- All of the radar rainfall estimation algorithms will give inaccurate answers when formulated or calibrated using data corresponding to a different storm structure than what occurs during validation.
- The Kurnell radar Z-R relationships are different for different storm types and may vary within the same storm type.

6. FURTHER WORK

The further stages of this work are to estimate a transformed, hence unbiased, radar reflectivity by using appropriately formulated parametric relationships between observed reflectivity and physical nature of storms. After we obtain the unbiased reflectivity, the final radar rainfall will be estimated by matching their CDF to the CDF of raingauge rainfall. Results from this work will be presented at a later date.

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